

# EXTRA DIMENSIONS AND SELF-ORGANIZING CRITICALITY <sup>a</sup>

A.B. KOBAKHIDZE

*HEP Division, Department of Physics, University of Helsinki and  
Helsinki Institute of Physics, FIN-00014 Helsinki, Finland  
E-mail: Archil.Kobakhidze@Helsinki.fi*

We discuss a possible explanation of the hierarchy problem within the theories with spacetime dimensions higher than four. We show that the presence of relatively (not hierarchically) large extra dimensions can significantly alter the evolution of the Higgs field VEV, driving it to an infrared stable fixed point  $\sim M_W$ . Such a behaviour results in self-organizing criticality and naturally explains gauge hierarchy without any fine tuning of the parameters.

## 1 Introduction

During the last few years a new explanations to the familiar hierarchy between the fundamental high energy scales (say, the Planck scale  $M_{Pl} \approx 10^{18}$  GeV) and the electroweak scale  $M_W \approx 100$  GeV have been proposed within the high dimensional theories <sup>2,3</sup>. These scenarios for solving the hierarchy problem are radically different from those usually attributed to the supersymmetry or to the dynamical symmetry breaking and explore the fact that the mass scales can be significantly altered due to the geometry of extra space. The scenario of Refs. <sup>2</sup> utilize  $\delta$  extra compact dimensions with large compactification radii  $r_n$  ( $n = 1, \dots, \delta$ ) in the factorizable,  $M^4 \times N^\delta$ ,  $(4+\delta)$ -dimensional spacetime where all known particles and interactions (except the gravity) are localized on a four-dimensional hypersurface  $M^4$  (3-brane). Assuming then that the fundamental high-dimensional scale  $M_*$  is just an order of magnitude or so larger than the electroweak scale  $M_W$ , the apparent weakness of gravity (or as it is the same, the heaviness of the Planck mass  $M_{Pl}$ ) in the visible four-dimensional world ( $M^4$ ) is explained due to the large volume  $V_{N^\delta}$  of the extra-dimensional submanifold  $N^\delta$ :

$$M_{Pl}^2 = M_*^{\delta+2} V_{N^\delta}. \quad (1)$$

The scenario of Ref. <sup>3</sup> deals with a 5-dimensional non-factorizable  $AdS_5$  spacetime where two 3-branes located at the  $S^1/Z_2$  orbifold fixed points of the fifth

---

<sup>a</sup>Based on the talks given at ICTP Conference on "Physics Beyond Four Dimensions", 3-6 July 2000, Trieste, Italy, and at ISPM Workshop "Modern trends in Particles and Cosmology", 10-15 September 2000, Tbilisi, Georgia

compact dimension. Now the weakness of gravity in the visible world 3-brane is explained without recourse to large extra dimensions, but rather as a result of gravity localization on the hidden 3-brane. Gravity localization in such scenario occurs because the five-dimensional Einstein's equations admit the solution for the space-time metric with a scale factor ("warp factor") which is a falling exponential function of the distance along the extra dimension  $y$  perpendicular to the branes<sup>b</sup>

$$ds^2 = e^{-2k|y|} dx_{1+3}^2 + dy^2, \quad (2)$$

Thus, graviton is essentially localized on the hidden brane with positive tension which is located at  $y = 0$  fixed point of the  $S^1/Z_2$  orbifold, while the Standard Model particles are assumed to be restricted on the visible brane with negative tension which is located at  $y = \pi r_c$  ( $r_c$  is the size of extra dimension) orbifold fixed point. So, a hierarchically small scale factor generated for the metric on the visible brane gives an exponential hierarchy between the mass scales of the visible brane and the fundamental mass scale  $M_*$ , after one appropriately rescales the fields on the visible brane.

The crucial point of any successful solution of the hierarchy problem is a stability of the hierarchy under the radiative corrections. For the scenarios described above, the question of quantum stability is translated to the problem of the stability of radii of extra dimensions. Indeed, while the scenario of Refs. <sup>2</sup> does eliminate the Planck/weak scale hierarchy, it introduces a new the same order hierarchy between  $\mu_c$  ( $\mu_c = 1/R_c$ , where  $R_c$  is a radius of extra dimensions) and  $M_W$ ,  $\frac{\mu_c}{M_W} \approx \frac{M_W}{M_{Pl}}$ <sup>2</sup>. Thus the stability of large extra dimensions remains as a critical question. The radius stabilization is also crucial for the scenario of Ref. <sup>3</sup>, although the hierarchy needed in this case is relatively small,  $\mu_c/M_{Pl} \simeq 10^{-1} \div 10^{-2c}$ . Several proposals to solve this problem have been discussed in the literature (for some of them, see <sup>6</sup>).

Here we would like to suggest an alternative mechanism for the solution of gauge hierarchy problem in higher dimensional theories with relatively large radii extra dimensions <sup>7</sup>. We explore an old idea of "self-organizing criticality" proposed in Refs. <sup>8d</sup>. The idea is the following: The electroweak scale  $M_W$  in the SM originates from the spontaneous electroweak symmetry breaking and thus

---

<sup>b</sup>Another remarkable thing offered by such a solutions is the possibility to reproduce the four-dimensional Newton's law in our universe even with infinitely large (non-compact) extra dimensions <sup>4,5</sup>.

Another deficiency of this model is the *ad hoc* fine tuning required between the cosmological constants in the bulk and on the 3-branes, in order to obtain the desired solution (2). The stability of such fine tuning seems also to be problematic.

Basically, the concept of self-organizing criticality where a certain dynamical systems drive themselves to a critical state over the wide range of length and time scales has been

essentially determined by the vacuum expectation value (VEV) of the scalar doublet field  $\langle \phi \rangle \approx 174$  GeV ( $M_W \sim \langle \phi \rangle$ ) which is an order parameter for the electroweak phase transition. So the question why the electroweak scale is so small compared with the fundamental high energy scales can be reformulated: Why is the SM near the phase transition?, or: Why is the system near criticality? A natural solution to the hierarchy problem then is possible when the system is in a situation of self-organizing critically, i.e. when it is near criticality not only for a particular tuning of theory parameters (scalar mass, self-interaction coupling, etc...) at high energies, but for a wide range of them. Thus the self-organizing criticality is possible if there are an infrared fixed-points (at least approximate) in the evolution of a certain parameters and is closely related to large anomalous dimensions <sup>8</sup>. Indeed, let us consider the evolution equation for the renormalized VEV  $v_R$  of the scalar field:

$$\frac{dv_R^2(\mu)}{d(\ln \mu)} = A v_R^2(\mu), \quad (3)$$

where  $A$  is an anomalous dimension which generally depends on the running parameters of the theory (self-interaction couplings, scalar-fermion Yukawa couplings, gauge couplings, etc.) and  $\mu$  is an energy scale. In the limit of constant  $A$  the solution to (3) can be easily found:

$$\frac{v_R^2(M_W)}{v_R^2(M_{Pl})} = \left( \frac{M_W}{M_{Pl}} \right)^A \quad (4)$$

It is evident from (4) that for  $A = 2$  the ratio  $\frac{v_R^2(\mu)}{\mu^2}$  exhibits infrared stable fixed point behaviour, that, even for a naturally expected large values of the VEV at high energies  $v_R^2(M_{Pl}) \simeq M_{Pl}^2$ , could lead to a large desired hierarchy at low energies.

However in four dimensions,  $A \geq 2$  is highly undesirable, since anomalous mass dimension  $A$ , being proportional to coupling constants, is usually  $\ll 1$ , unless some of the couplings (Higgs self-interacting coupling or/and Yukawa couplings) are non-perturbative below the scale  $M_{Pl}$ , or there is an unrealistically large number of degrees of freedom ensuring  $A \geq 2$  <sup>8</sup>. Needless to say, that it is very difficult (if ever possible) to construct a realistic model obeying such conditions. In higher dimensional theories, however, the situation is drastically changed. The point is that, owing to the power-law (in contrast to the logarithmic in four dimensions) evolution of the theory parameters, the

---

introduced in Ref.<sup>9</sup> and subsequently applied to a various systems starting from the multi-scale structure of the natural world and ending by the economic and living systems (see e.g. Ref.<sup>10</sup> and references therein).

Higgs vacuum expectation value (VEV), while being of the order of  $M_{Pl}$  at high energies, rapidly decrease down to the infrared stable fixed point  $\sim M_W$  even for the small values of  $A$ , thus naturally inducing a large hierarchy even in the case of SM with the ordinary number of colours and flavours.

## 2 A toy model

To be more quantitative, let us now a simplified example of the  $SU(N)$ -symmetric Higgs-Yukawa system with  $N_c$  colours <sup>7</sup>. Our starting action in  $D = 4 + \delta$  dimensions ( $\delta$  is the number of extra compact dimensions) is

$$\begin{aligned} S_{\Lambda_0} = & \int d^{4+\delta} [Z(\Lambda_0) \partial_\mu \phi^\dagger \partial^\mu \phi - \mu^2(\Lambda_0) \phi^\dagger \phi + \frac{1}{2} \lambda(\Lambda_0) (\phi^\dagger \phi)^2 \\ & + Z_L(\Lambda_0) \bar{\psi}_L i \gamma_\mu \partial^\mu \psi_L + Z_R(\Lambda_0) \bar{\psi}_R i \gamma_\mu \partial^\mu \psi_R \\ & + (h(\Lambda_0) \bar{\psi}_L \phi \psi_R + h.c.)], \end{aligned} \quad (5)$$

where  $\phi$  is an  $N$ -component complex scalar field,  $\psi_L$  is an  $N$ -component left-handed fermion field with  $N_c$  colours and  $\psi_R$  is a right-handed  $SU(N/2)$ -singlet fermion with  $N_c$  colours again.  $Z$ ,  $Z_L$ , and  $Z_R$  in (1) are the field renormalization factors which we choose to be equal to 1 at the scale  $\Lambda_0$ . In the case of  $N = 2$  and  $N_c = 3$  the action (1) is just the SM action in the limit of vanishing gauge couplings and Higgs-Yukawa couplings except for one type of quarks.

Theory with action (1) in higher dimensions ( $\delta \neq 0$ ) is known to be non-renormalizable, but it can be well defined by introducing an ultraviolet cut-off  $\Lambda_0$ , which is natural to identify with the fundamental Planck scale  $M_{Pl}$ . At low energies one can consistently describe the theory using Wilsonian effective action approach <sup>11</sup>. The basic idea behind this approach is first to integrate out momentum modes between a cut-off scale  $\Lambda_0$  and lower energy scale  $\Lambda$ , rather than to integrate over all momentum modes in one go. The remaining integral from  $\Lambda$  to zero may again be expressed as a partition function, but the bare action  $S_{\Lambda_0}$  (5) is replaced by a complicated effective action  $S_\Lambda$  (Wilsonian effective action) and the overall cut-off  $\Lambda_0$  by the effective cut-off  $\Lambda$ , in such a way that all physics, i.e. Green functions, are left invariant. The difference in  $S_\Lambda$  induced by the change of the cut-off is determined integrating "shell modes" with momenta between  $\Lambda$  and  $\Lambda + \delta\Lambda$  and for an infinitesimal  $\delta\Lambda$  becomes a Gaussian path integral which can be exactly carried out. Thus, the scale dependence of the Wilsonian effective action is given by the exact functional differential equation

$$\Lambda \frac{\partial S_\Lambda}{\partial \Lambda} = \mathcal{O}[S_\Lambda], \quad (6)$$

where  $\mathcal{O}[S_\Lambda]$  is a non-linear operator acting on the functional  $S_\Lambda$ . However, for the practical calculations it is inevitable to approximate the evolution equation (6). We have adopted here so-called local potential approximation<sup>12</sup> with a sharp cut-off and have truncated the effective potential keeping only renormalizable terms up to  $\phi^4$ . In this approximation, defining the effective renormalized four-dimensional VEV  $\bar{v}$  self-interaction  $\bar{\lambda}$  and Higgs-Yukawa  $\bar{h}$  couplings through the five-dimensional renormalized parameters  $v_R$ ,  $\lambda_R$  and  $h_R$ , respectively, as<sup>e</sup>

$$\bar{v}^2 = (2\pi R_c)^\delta v_R^2, \quad \bar{\lambda} = (2\pi R_c)^{-\delta} \lambda_R, \quad \bar{h} = (2\pi R_c)^{-\frac{\delta}{2}} h_R, \quad (7)$$

we obtain eventually the following evolution equations (for more details see Ref. 7):

$$\Lambda \frac{d\bar{v}^2}{d\Lambda} = \frac{K_D}{2} \left( \frac{2\pi\Lambda}{\mu_c} \right)^\delta \left[ -6\bar{\lambda} - \frac{2^{\frac{D}{2}+1}}{D} N_c \bar{h}^2 + 2^{\frac{D}{2}} N_c \frac{\bar{h}^4}{\bar{\lambda}} \right] \bar{v}^2, \quad (8)$$

$$\Lambda \frac{d\bar{\lambda}}{d\Lambda} = \frac{K_D}{2} \left( \frac{2\pi\Lambda}{\mu_c} \right)^\delta \left[ (2N+8)\bar{\lambda}^2 + \frac{2^{\frac{D}{2}+2}}{D} N_c \bar{h}^2 \bar{\lambda} - 2^{\frac{D}{2}} N_c \bar{h}^4 \right], \quad (9)$$

$$\Lambda \frac{d\bar{h}^2}{d\Lambda} = K_D \left( \frac{2\pi\Lambda}{\mu_c} \right)^\delta \frac{2(N+1) + 2^{\frac{D}{2}} N_c \bar{h}^4}{D} \bar{h}^2 \quad (10)$$

where  $K_D = \frac{2^{1-D}}{\pi^{-D/2}\Gamma(D/2)}$  is the  $D$ -dimensional angular integral. Note, that by taking  $\delta = 0$  the set of Eqs. (8-10) correctly reproduces the familiar one-loop results of perturbation theory in four dimensions. The crucial role of the extra dimensions in solving the gauge hierarchy problem can be seen from Eqs. (8-10) even without performing numerical calculations. Indeed, ignoring for the moment the running of  $\bar{\lambda}$  and  $\bar{h}$ , one finds from (8)

$$\frac{\bar{v}(M_W)}{\bar{v}(M_{Pl})} = \left( \frac{M_W}{\mu_c} \right)^{\frac{\omega_0}{2}} \exp \left[ \frac{(2\pi)^\delta}{2\delta} \omega_\delta \left( 1 - \left( \frac{M_{Pl}}{\mu_c} \right)^\delta \right) \right], \quad (11)$$

where  $\omega_\delta = \frac{K_D}{2} \left[ -6\bar{\lambda} - \frac{2^{\frac{D}{2}+1}}{D} N_c \bar{h}^2 + 2^{\frac{D}{2}} N_c \frac{\bar{h}^4}{\bar{\lambda}} \right]$ . The exponential factor in (11) can be naturally small in the case of extra dimensions ( $\delta \neq 0$ ) even for small (but positive) values of  $\omega_\delta$ , providing the desired hierarchy  $\frac{\bar{v}(M_W)}{\bar{v}(M_{Pl})} \approx$

---

<sup>e</sup> Here we assume that  $\delta = D - 4$  extra dimensions are compactified on a circle of a fixed radius  $R_c = \frac{1}{\mu_c}$ . The factor  $(2\pi R_c)^\delta$  is just the volume of extra space appeared in the effective four-dimensional action after one integrates over the extra space.

$\frac{M_W}{M_{Pl}}$ , while in four dimensions this ratio is of the order of  $O(1 \div 10)$  unless  $\omega_0 = A_0 \geq 2$ , that can, however, never be obtained in perturbation theory since for small couplings  $A_0$  is proportional to these couplings<sup>8</sup>, as already discussed above.

Of course, the actual solutions of the set of Eqs. (8-10) is more complicated, since the Yukawa and self-interaction couplings also exhibit fast (power-law) running and the approximation of the constant  $\bar{\lambda}$  and  $\bar{h}$  is very crude. We have analyzed Eqs. (8-10) numerically. The Yukawa coupling  $\bar{h}$  rapidly decreases going down in the energy region between  $M_{Pl}$  and  $\mu_c$  and drives to the infrared stable fixed-point  $\bar{h} = 0$ . If the Yukawa coupling dominates over the  $\bar{\lambda}$  ( $\bar{h}^2 \gg \bar{\lambda}$ ) then  $\bar{\lambda}$  at the same time increases for smaller energies, until  $\bar{\lambda}$  becomes large enough so that the terms proportional to  $\bar{\lambda}^2$  and  $\bar{h}^2 \bar{\lambda}$  cancel the term proportional to  $\bar{h}^4$  in (9). Thus,  $\bar{\lambda}$  approaches the infrared stable fixed-point,  $\bar{\lambda} \sim \bar{h}^2$ . At the same time, even starting with large initial values of  $\bar{v}(M_{Pl}) \lesssim M_{Pl}$ ,  $\bar{v}$  rapidly decreases and below the  $\mu_c$  changes very slowly. Thus, for certain  $\mu_c$  and  $\delta$  the mean value of anomalous dimension  $A_\delta$  can be equal to 2, which means that  $\frac{v^2(\Lambda)}{\Lambda^2}$  has an infrared stable quasi fixed-point. Indeed, we have explicitly checked by solving numerically the system of Eqs. (8-10), that the ratio  $\frac{\bar{v}(\bar{v})}{\bar{v}(M_{Pl})}$  is actually stable under the variation of the scalar VEV at Planck scale with  $\bar{\lambda}(M_{Pl})$  and  $\bar{h}(M_{Pl})$  fixed. For example, for  $\bar{\lambda}(M_{Pl}) = 0.2$ ,  $\bar{h}(M_{Pl}) = 3$ ,  $\mu_c = 10^{16.75} \text{GeV}$ ,  $\delta = 1$  we obtain for the average value of the anomalous dimension between the scales  $M_{Pl}$  and  $\bar{v}$

$$\langle A \rangle \equiv (\ln \frac{\bar{v}}{M_{Pl}})^{-1} \int_{\ln \frac{\bar{v}}{M_{Pl}}}^0 A_\delta(\Lambda) d(\ln \frac{\Lambda}{M_{Pl}}) \approx \frac{2}{1 + 0.03 \ln \frac{M_{Pl}}{\bar{v}(M_{Pl})}}. \quad (12)$$

So, if  $\bar{v}(M_{Pl}) \approx M_{Pl}$ , as it is naturally expected,  $\langle A \rangle$  is close to 2, providing large stable hierarchy  $\frac{\bar{v}(\bar{v})}{\bar{v}(M_{Pl})} \approx 1.8 \cdot 10^{-15}$ . Thus, varying  $\bar{v}(M_{Pl})$  by 10% around  $10^{17} \text{GeV}$ , we obtain  $\langle A \rangle \approx 2.14 \div 2.13$  and  $\bar{v}(\bar{v}) = 157 \div 190 \text{ GeV}$ . Furthermore, requiring that  $\bar{\lambda}$  and  $\bar{h}$  are within the perturbative regime ( $\frac{\bar{\lambda}^2}{4\pi} < 1$  and  $\frac{\bar{h}^2}{4\pi} < 1$ ) and that the relations (8-10) hold for the whole interval between  $M_W$  and  $M_{Pl}$ , our toy model predicts the upper bounds on the physical masses of the scalar and fermion, respectively:

$$m_S \lesssim 73 \text{GeV}, \quad m_F \lesssim 100 \text{GeV}. \quad (13)$$

It should be stressed that our solution to the gauge hierarchy problem does not require the extra dimensions to be large. In fact, the hierarchy  $\frac{\mu_c}{M_{Pl}} \sim 0.05 \div 0.3$  is enough to get the desired values of  $\bar{v}$  at low energies, even starting

with naturally expected large values of  $\bar{v}$  at  $M_{pl}$  ( $\bar{v}(M_{pl}) \sim M_{Pl}$ ). Indeed, requiring that  $\bar{v}(174 \text{ GeV}) = 174 \text{ GeV}$  and  $\bar{\lambda}(174 \text{ GeV}) = 0.25$ ,  $\bar{h}(174 \text{ GeV}) = 0.55$ , we have obtained numerically  $\mu_c = 10^{16.75}$ ,  $10^{17.30}$ ,  $10^{17.51} \text{ GeV}$  for  $\delta = 1, 2$ , and  $3$ , respectively.

### 3 Conclusions and outlook

We have discussed here a possible explanation of the hierarchy problem within the theories with spacetime dimensions higher than four. We have shown that the presence of relatively (not hierarchically) large extra dimensions can significantly alter the evolution of the Higgs field VEV, driving it to an infrared stable fixed point  $\sim M_W$ . Such a behaviour results in self-organizing criticality and naturally explains gauge hierarchy without any fine tuning of the parameters.

Let us conclude with the following comments. It is clear, that more accurate calculations, related to an improved treatment of thresholds and contributions beyond the LPA approximation and truncation of the effective potential as well as higher loop corrections quantitatively alter (perhaps quite significantly) our predictions for particle masses in (13), but qualitatively the behaviour of the parameters seems to remain unchanged, thus providing us with a the natural solution of the gauge hierarchy problem as discussed above.

The idea discussed here can be also applied to explain other observed hierarchies. The possible role of the fixed point solutions in generating the fermion mass hierarchy within the high-dimensional theories have been already discussed in<sup>12</sup>. It is interesting to investigate whether or not this mechanism can be applied for the solution to the cosmological constant problem as well<sup>13</sup>. Finally, while we have demonstrated our mechanism for the solution of the gauge hierarchy problem on the simplified model of Higgs-Yukawa interaction, we find no reason why it can not work in the realistic models when a full set of the particles and forces of the SM or its extensions will be included. Moreover, several examples of nonsupersymmetric unification through extra dimensions have been presented<sup>14</sup>, so we believe that a unified model without the gauge hierarchy problem and consistent with present experimental data can be constructed.

### Acknowledgements

This work was supported by the Academy of Finland under the Project No. 163394. ABK would like also to gratefully acknowledge the support from

ICTP and the Chancellor of University of Helsinki in attending the Conference  
"Physics Beyond Four Dimensions", Trieste, 2000.



## References

- 1.
2. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263; Phys. Rev. D59 (1999) 086004;  
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257.
3. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
4. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999);  
J. Lykken and L. Randall, hep-th/9908076.
5. M. Gogberashvili, hep-ph/9812296; Europhys. Lett. **49**, 396 (2000); Mod. Phys. Lett. **A14**, 2025 (1999); hep-ph/9908347.
6. N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-th/9809124;  
K. R. Dienes, E. Dudas, T. Gherghetta and A. Riotto, Nucl. Phys. **B543**, 387 (1999);  
Z. Kakushadze, Phys. Lett. **B466**, 153 (1999);  
W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999);  
M. A. Luty and R. Sundrum, hep-th/9910202.
7. M. Chaichian and A. B. Kobakhidze, Phys. Lett. **B478**, 299 (2000).
8. C. Wetterich, Phys. Lett. **B104**, 269 (1981);  
S. Bornholdt and C. Wetterich, Phys. Lett. **B478**, 299 (1992).
9. P. Bak, C. Tang and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987);  
Phys. Rev. **A38**, 3645 (1988).
10. M. Creutz, hep-lat/9611018.
11. K. G. Wilson and I. G. Kogut, Phys. Rep. **12C** 75 (1974);  
F. J. Wegner and A. Houghton, Phys. Rev. **A8** (1973) 401;  
J. Polchinski, Nucl. Phys. **B231** 269 (1984); A. Ringwald and C. Wetterich, Nucl. Phys. **B334** 506 (1990).
12. K. R. Dienes, E. Dudas and T. Gherghetta, Nucl. Phys. **B537** 47 (1999);  
S. A. Abel and S. F. King, Phys. Rev. **D59** 095010 (1999);  
A. B. Kobakhidze, hep-ph/9904203 (to appear in Phys. Atom. Nucl.);  
K. Yoshioka, Mod. Phys. Lett. **A15** 29 (2000);  
M. Bando, T. Kobayashi, T. Noguchi and K. Yoshioka, Phys. Lett. **B480** 187 (2000).
13. M. Chaichian and A. B. Kobakhidze, work in progress.
14. K. R. Dienes, E. Dudas and T. Gherghetta in Ref. <sup>12</sup>;  
P. H. Frampton and A. Rasin, Phys. Lett. **B460** 460 (1999);  
G. K. Leontaris and N. D. Tracas, Phys. Lett. **B454** 53 (1999);  
A. Perez-Lorenzana and R. N. Mohapatra, Nucl. Phys. **B559** 255 (1999).